Electricity Prices: Stochastic or Deterministic?

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***Abstract*—In this paper (spot) electricity prices are analyzed. Fluctuations in electricity prices - as they come about on the new energy markets - turn out to be deterministic to a large extent. This is very different from (spot) market prices for stocks, currencies, interest or common commodities, like wheat or oil, which are best treated as pure stochastic variables. Therefore the (Geometric) Brownian Motion Price Theory - as is commonly used for stocks and other markets - does not work for electricity. Instead a much simpler and better descriptive Market Based Price Forecasting methodology is proposed.**

***Index Terms*—Electricity Prices, Energy Risk Management, Gaming, (Geometric) Brownian Motion and Jumps.**

I. INTRODUCTION

ONE of the foundations of commodity price theory are the papers by Fisher Black & Myron Scholes and Robert Merton, both in 1973 on option pricing [1,2]. In these papers they use a closed form solution for the so-called classic Geometric Brownian Motion stochastic differential equation (GBM sde) [3] which seems to describe best the behavior of daily stock market prices.

This basic idea was then taken for the emerging financial and commodity markets. Even though daily prices of the latter markets do not seem to be governed by the basic (Geometric) Brownian Motion stochastic differential equation. This lead to modifications of which the Mean Reversion process (Ornstein-Uhlenbeck process) is widely accepted [4]. World financial and commodity markets have now grown to billions or even trillions of dollars, traded daily. It can be said that this was only possible because of the mathematical sophistication to 'predict' (spot) market prices [5].

However, many times these models have shown to break down at tremendous costs but still people keep using them because having a model is better than having no model at all [6, 7].

With the introduction of electricity markets in the 1990s the need to model electricity prices arose, as opposed to state regulation, which had become the norm since the introduction of large scale electricity production and consumption in the beginning of the 20^{th} century [8]. Although electrical energy cannot be stored in large quantities, the well known GBM sde (now modified to include so-called jumps or spikes) is again used to describe the pricing process for (spot) electricity. Even though the cost-of-carry arbitrage argument, which implies storability of the commodity (a requirement electricity cannot fulfill on the scale needed [9]), is the main underlying assumption of Brownian Motion.

II. (GEOMETRIC) BROWNIAN MOTION

(Geometric) Brownian Motion on stock spot prices S(t), has the following form:

$$dS(t) = \mu.S(t)dt + \sigma.S(t)dW(t) \qquad \text{(Eq. 1)}$$

with: μ the mean and σ the standard deviation (or volatility) of the spotprice. The time step dt is usually a day and dW(t) is a stochastic variable (Wiener process) with a standard normal distribution.

To describe a commodity price path, such as natural gas, Mean Reversion (Ornstein-Uhlenbeck process) or the tendency to return to an average spot price has to be included:

$$dS(t) = \alpha.\{\mu - \ln[S(t)]\}.S(t)dt + \sigma.S(t)dW(t) \qquad \text{(Eq. 2)}$$

with: α the Mean Reversion Rate, for some gas markets this number is around 10 [4], which means that the half-time of a price move dS away from the mean is around 25 days.

To describe electricity prices, yet another modification of the (Geometric) Brownian Motion sde is needed; so-called jumps or spikes:

$$dS(t) = \alpha.\{\mu - \ln[S(t)]\}.S(t)dt + \sigma.S(t)dW(t) + \kappa.S(t)dP(t) \qquad \text{(Eq. 3)}$$

with: κ the jump-amplitude and dP(t) the stochastic process describing the frequency of the jumps (usually a Poisson process).

As can be seen from fig. 1 for Dutch APX 2001 Day-Ahead Market (DAM) Prices in Eur/MWh, the jump-regime is to a large extent governing the price path. For a description of the Dutch APX DAM please refer to the appendix.

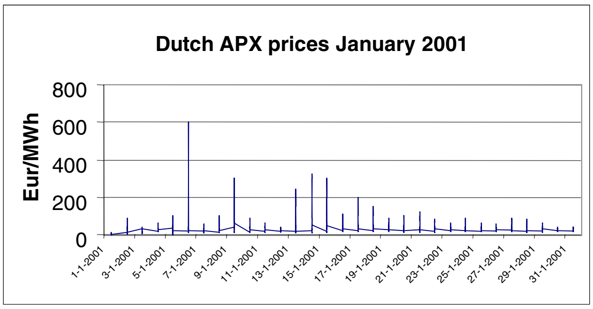

Fig. 1 Dutch APX DAM Price January 2001

Because of the jumps, modified closed form solutions for equations 1 and 2 - as are frequently used [7] - are not suited for electricity markets and apart from theoretical progress to (partly) analytically solve equation 3 [10], numerical solutions provide the best answers.

All solutions of equation 3 require an estimation of the mean μ, volatility σ, Mean Reversion Rate α and jump characteristics κ and dP. Although μ can usually be observed from the market in form of forward prices, the σ, α, κ and dP are not easily observable and their (least-squares) historical estimations, such as Recursive Filtration [4], are greatly influenced by jumps.

III. MEDIAN ABSOLUTE DEVIATION

The main problem with linear regression or least-squares estimation, as are almost solely used to analyze historical price (returns) series to find σ and α, is their sensitivity to outliers.

Estimators like the mean and the variance are highly influenced by outliers because of what is called their very low finite sample breakdown point (BP), i.e. the smallest proportion of observations in a (time) series that can result in the sample mean being arbitrarily large or small. As an example consider the series {X} = {2, 3, 4, 5, 6, 7, 8, 9, 10, 50}. The estimator for the population mean is the sample mean μ_s = 10.4 and the sample standard deviation amounts to s = 14.15.

If we declare outliers by the rule: $|X - \mu_s| > 2s$, we see that 50 is an outlier. However, if we change the number 10 also to 50, the series suddenly contains no outliers. If we increase those two 50s to 100 or even 1000 still both would not be flagged as outliers! This is because the breakdown point for the mean of a series with n observations is 1/n; one observation can change the sample mean to a number anywhere between -∞ and +∞.

A much better way to declare outliers is by using the median: M [11]. Using the Median Absolute Deviation or MAD statistic, to find outliers according to the rule: $| X – M | > 2.(MAD/0.6745)$, produces much better results. This is because the finite sample breakdown point of the median is 0.5, i.e. 50% of observations have to be changed to alter the median. The MAD is defined as the median of the series: $|X_1-M|$, $|X_2-M|$, $|X_3-M|$, … , $|X_n-M|$. Using this method we immediately recognize the two 50s, 100s or 1000s as outliers and discard them from further analysis.

Price series must have quotes for each moment in time. An MAD filtered price series can be made to agree with this requirement by replacing the outlier (or jump) by its border value: M ± 2.(MAD/0.6745). This creates a price series, which is best described as lognormally distributed (see fig. 7 & 8).

IV. REGIMES IN ELECTRICITY PRICES

Because of the nature of electro-magnetic waves, electricity cannot be stored in large quantities. Consequently supply and demand must be in balance every moment in time. Therefore the (spot) market electricity prices follow the expected demand.

For each daily load cycle at first the generation capacity with the lowest quoted price (usually based on the marginal costs) is dispatched. When the demand rises - during the (early) morning hours - more expensive capacity is put online. When the demand declines again - during the late afternoon and evening hours - the more expensive peaking units are put out of service directly followed by the mid-range units.

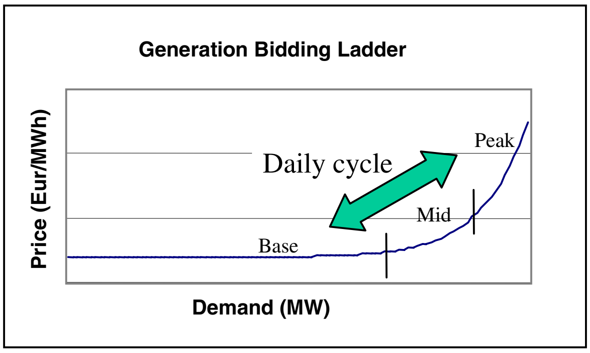

Fig. 2 Stylized generation bidding ladder

This standard pattern repeats itself every day (fig. 2) only to be disrupted by unexpected situations such as a tie-line (interconnector) outage or tripping of a large generator or – as happens in (emerging) free markets – by rumors. In these cases (very) expensive peaking units have to be started and put online to keep the physical balance. The price of these units is much higher when compared to the more frequently used units and, because the situation allows so, gaming starts to be a significant factor.

Two market regimes or states can be distinguished:

1. the normal or equilibrium regime: the daily load cycle and generation capacity develop as expected;
2. the abnormal or non-equilibrium regime: a situation in which - if nothing would be done - an imbalance (usually a

generation shortage) can occur and strong price impulses have to be given to stimulate the market participants to make the necessary moves.

Regimes are not specific to the Dutch market but have been recognized before, e.g. the Californian market [12]. In fact, regimes have been incorporated in Brownian Motion models by so-called Markov regime switching, i.e. there is a certain probability that the price solution space switches from one state to the next [13].

V. Analysis of Demand

Since demand – in relation to available generation capacity - drives the (spot) market electricity price, an analysis of national or market load characteristics - especially seasonality - is required. When we consider the Dutch national load in 2001, fig. 3 and 4, we can observe that the load or demand characteristics on working days (day 1 to 5) are similar. The behavior on Saturdays (day 6) starts to deviate, whereas Sundays (day 7) are consistently different. So we can recognize both a daily (each hour in a day) seasonality and a weekly (each day in a week) seasonality.

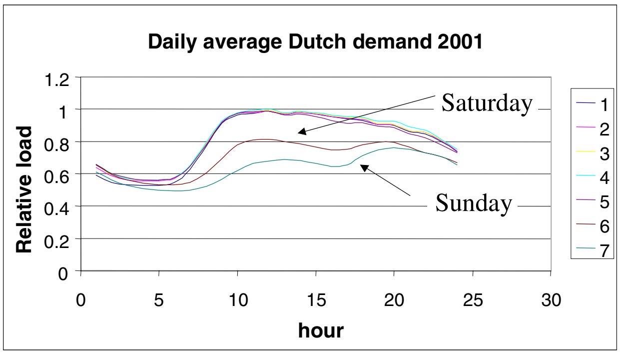

Fig. 3 Average daily Dutch demand 2001

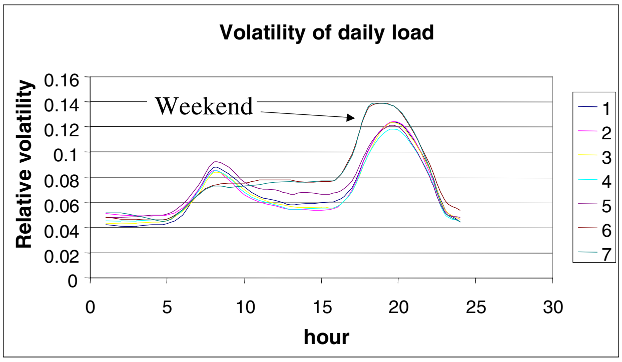

Fig. 4 Standard deviation daily Dutch demand 2001

VI. Analysis of Dutch APX prices 2001

On the Dutch APX market mainly two different products, on-peak blocks (weekdays 07:00–23:00h) and off-peak blocks (weekdays 23:00–07:00h plus weekends), are traded.

Analysis of the APX price series of 2001 by means of the MAD filtration methodology, as discussed in section III, shows that the underlying equilibrium regime has prices varying roughly between 15 and 55 Eur/MWh for on-peak and between 10 and 25 Eur/MWh for early morning off-peak hours (see fig. 5 & 6).

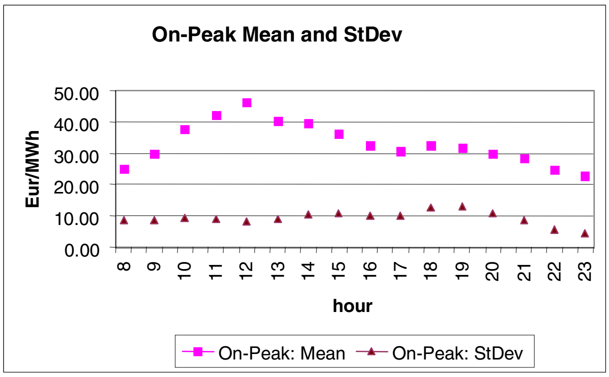

Fig. 5 On-peak prices (equilibrium regime)

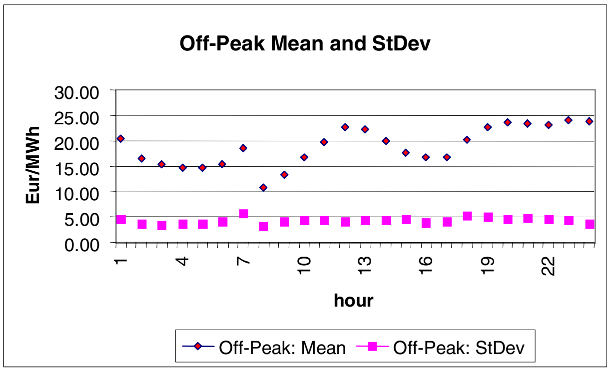

Fig. 6 Off-peak prices (equilibrium regime)

From fig. 5 and 6 we see that filtered APX prices neatly follow the average demand curves as shown in fig. 3. Note that for off-peak hours (23:00-07:00) the price follows the Saturday and Sunday demand curves (day 6 & 7). Because of this similarity we apply a regression analysis between a normalized national load profile (2001) and a normalized MAD-filtered APX price series (2001) for the on-peak and off-peak price-bands. For reasons of similarity we will treat Saturday as a weekday. The normalization is performed by dividing each hourly load or price by the daily average. The resulting MAD-filtered price histograms are shown in fig. 7 & 8.

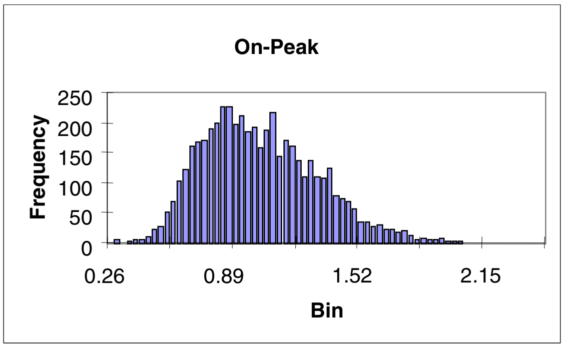

Fig. 7 Normalized on-peak price histogram

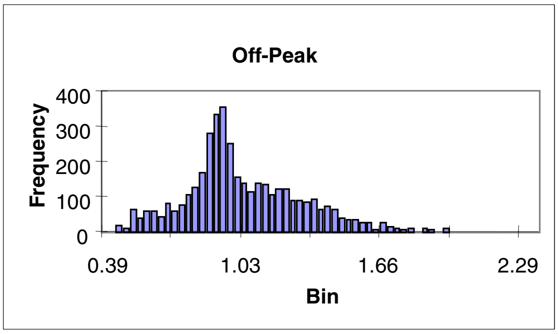

Fig. 8 Normalized off-peak price histogram

These histograms clearly show that the usual 'fat tails' [4] in electricity price probability density functions (pdf's), as compared to the lognormal pdf, have been removed and that the remaining price series resemble the lognormal.

Analysis of the market-equilibrium regimes in fig. 9 and 10 shows that third order polynomials, for each of the price-bands, give rather high coefficients of determination: R^2-On = 0.72 and R^2-Off = 0.69. Other relationships, e.g. linear, power or exponential, resulted in lower coefficients of determination.

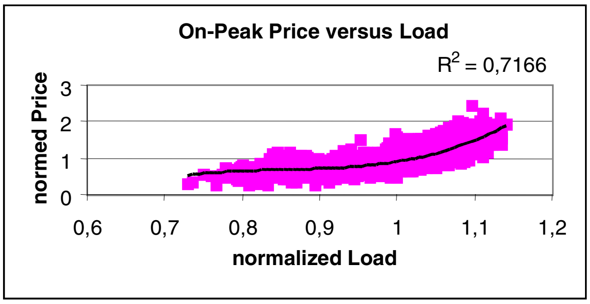

Fig. 9 On-peak price versus the normalized load

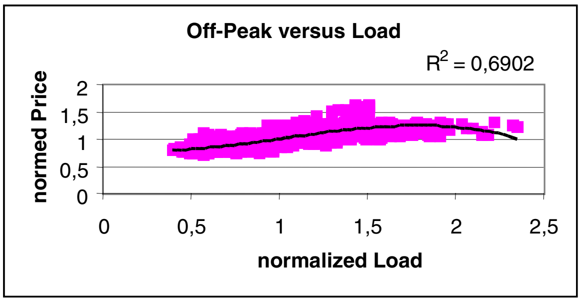

Fig. 10 Off-peak price versus the normalized load

A comparison of the equilibrium regimes of fig. 9 & 10 with the stylized bidding ladder in fig. 2 clearly shows that the MAD filtering captures the fundamentals of an electricity market. As expected the on-peak price has a tendency to increase with the load, whereas the off-peak price does quite the opposite for the upper range of the loads; overall the off-peak curve is rather flat. The horizontal leveling or even slight drop-off of the on-peak prices at lower loads and the horizontal leveling of the off-peak prices at the higher loads (Sunday evenings mostly) can both be explained by a must-run base capacity.

MAD filtration with x.(MAD/0.6745), in which x = 2, was used to make the division into an equilibrium and a non-equilibrium regime. Taking smaller multiples (x < 2) of the 'standard' deviation of the time series, i.e. (MAD/0.6745), did not produce much higher coefficients of determination (R^2). For higher multiples (x > 2.5) the on-peak and off-peak regressions start to decrease significantly.

We define jumps or spikes in the APX market as outliers found by the MAD filtration with x = 2. If we define four price-bands - weekday on-peak, weekday off-peak, Saturday and Sunday - it turns out that x = 1.5 gives the best fit for 3rd order polynomials with R^2 = 0.71; R^2 = 0.70; R^2 = 0.73 and R^2 = 0.71 respectively.

Analysis of the non-equilibrium regime, in tables 1 & 2 and fig. 11 through 14, shows that unexpected situations occur quite frequently. However, the frequency of known real problems with tie lines and/or generation capacity is much lower than the frequency of the jumps. This implies that gaming plays a significant role, as was analyzed through realistic market simulation for the Dutch power market [14]. Note that gaming is loosely defined as a non-equilibrium price level without a physical cause.

	Up	*Down*	*No Spike*	*Hours*
On-Peak	14,0%	0,2%	85,8%	4880
Off-Peak	9,2%	8,3%	82,4%	3880
Total	11,9%	3,8%	84,3%	8760

Table 1. Frequencies of jumps in the year 2001 APX prices.

μ-Mean	*Up*	*Down*	*No Spike*	
On-Peak	140,43	2,76	29,91	Eur/MWh
Off-Peak	38,59	3,09	17,84	Eur/MWh
σ-StDev	*Up*	*Down*	*No Spike*	
On-Peak	163,40	3,01	8,79	Eur/MWh
Off-Peak	25,02	3,00	4,00	Eur/MWh

Table 2. Amplitude characteristics of the APX 2001 jumps.

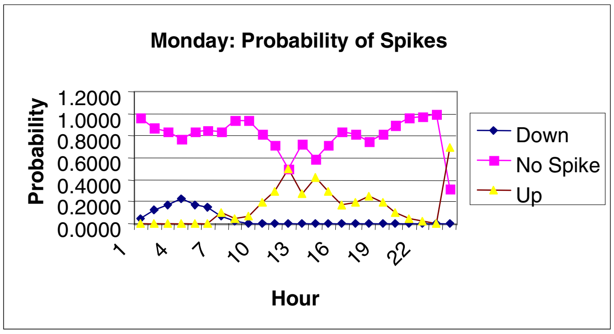

Fig. 11 Probability of jumps on Monday

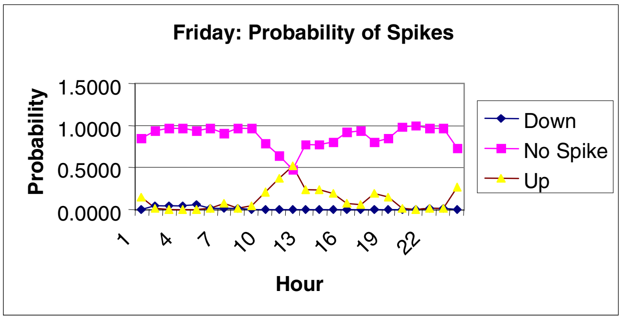

Fig. 12 Probability of jumps on Friday

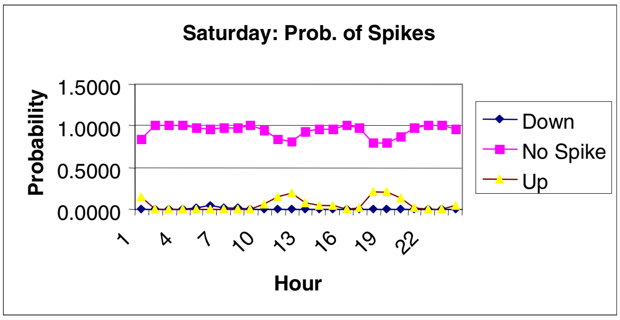

Fig. 13 Probability of jumps on Saturday

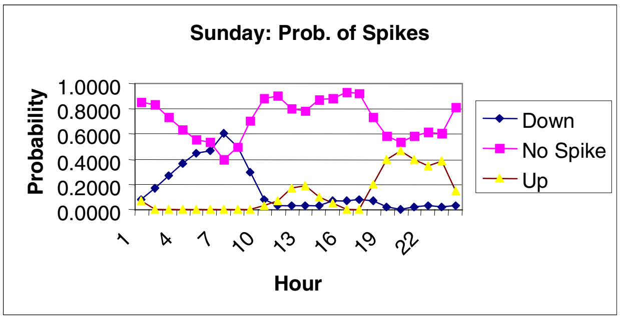

Fig. 14 Probability of jumps on Sunday

Note that Tuesdays through Thursdays are very similar to Fridays and are therefore not shown. From the figures we observe that gaming occurs at hours at which an overcapacity can be expected. Most notably at noon on weekdays (small) jumps occur with probabilities over 50%.

From fig. 11 it can be seen that Mondays have some potential for off-peak downward jumps in the early morning hours; this is not necessarily due to gaming, because of the APX block-bidding functionality (see appendix). On Sundays (fig. 14) downward jumps - prices drop to zero - are more or less the norm; in the Over The Counter (OTC) market, even negative prices can occur. Saturdays and Sundays are different from weekdays. On Saturdays jumps occur much less frequent. On Sundays the pattern is totally different with quite a high probability for upward jumps in the evening hours and a high propensity for downward jumps in the early morning hours.

When we compare these results with the average and volatility analysis of the total Dutch demand (national load) (fig. 3 & 4) most of this behavior can be explained.

The rise to (very) high probability levels for upward jumps during the morning ramp-up (see fig. 3) can be explained from the fact that a generation shortage can easily occur. This would immediately cause a physical unbalance and therefore market unbalances with proportionate costs. The much lower level of jumps on Saturdays can be explained from the fact that the load levels are well below those of weekdays - even though the volatility is higher - and therefore people do not get nervous so easily. Another aspect might be that many traderooms are simply closed or sparsely populated. On Sundays must-run capacity most certainly causes the many downward jumps during the early morning hours. The general rise of the load towards to the overall daily peak in the evening will, in combination with genco's having the possibility to start up (more) mid-range capacity, cause the many upward jumps during those hours.

The MAD analysis of the APX 2001 prices has been compared with an MAD analysis of the APX 2000 prices. In both years the frequency of upward jumps is comparable. Downward jumps did not occur in the year 2000 at all (Table 3). Regression analysis of the year 2000 normalized prices against the normalized load gives a low coefficient of determination for all functional relationships (R^2 between 0.3 and 0.4). This was caused by the so-called 'protocol' (i.e. a price agreement between genco's and disco's) causing the APX market in the year 2000 to be very thin. It was tried to game the market for its upward potential, but bidding was still very immature and not strongly related to demand.

	Up	*Down*	*No Spike*	*Hours*
On-Peak	12.1%	0.0%	87.9%	4860
Off-Peak	9.2%	0.0%	90.8%	3900
Total	10.8%	0.0%	89.2%	8760

Table 3. Frequencies of jumps in the year 2000 APX prices.

VII. New ways of modelling.

Next to (partly) analytical solutions to capture the specifics of jumps in (electricity) prices [10], a lot of effort is put in numerical or Monte Carlo solutions [15] of equation 3. The problem of historical estimates for the volatility σ, the mean reversion rate α and the characteristics of the jumps (κ and dP), to simulate possible price paths can be solved easily with the MAD filtering methodology.

The use of the mean reversion rate α to bring back a jump to normal price levels - as is standard practice - does not reflect the electricity price behavior very well (see also section VI). It is better to interpret α as the tendency of prices to return to an average price level during the non-jump regime and to consider the jumps as a separate process to be modeled by e.g. Markov switching [13]. This gives consistently better results and can be further improved if σ, α, κ and dP are determined after an MAD filtration.

Another approach is to describe the - to a large extent - deterministic aspects of the market by means of the relationships between the (national) load and the prices as determined for the equilibrium regime. The jumps can be added as a well-controlled stochastic process. Mathematically this Market Based Price Forecast (MBPF) looks like:

$$S_i(t) = Fw_i.\{g_i[\mu_L(t) + \sigma_L(t).dp] + J_i(t).dq\} \qquad \text{(Eq. 4)}$$

With; $S_i(t)$: the (spot) price for the different price-bands i, Fw_i: the forward for that period and price-band, g_i: the found functional relationship between price and load per price-band, $\mu_L(t)$: the average (predicted) load for the day at time t, $\sigma_L(t)$: the volatility of the load, dp: a stochastic variable $N(0,\sigma_g)$ [σ_g: accuracy of the load-relation], $J_i(t)$: jump amplitudes as found with the MAD filtering, and dq: a stochastic variable to describe the frequency of the jumps as found with the MAD filtering. Tuning the MBPF methodology is much more simple and intuitive than tuning equation 3, because the parameters of equation 4 are easily found by means of the MAD filtration and intuitively verifiable by traders. Because of the deterministic link to demand every fundamental aspect of the market [16], like all kinds of seasonality, is automatically included. Furthermore stochastic jumps can be included as they occurred in the past or perceived to occur in the future. This ensures a correct modeling of the gaming potential of the market.

When equation 4 is applied to generate price paths in order to determine simple call and put option premiums and the results are compared with those found by Monte Carlo simulation of the modified Brownian Motion model (equation 3), it appears that the proposed method produces very realistic price paths and equally good, but better understandable, option premiums.

However - as every model description of economic behavior - the new model does not provide the ultimate answer [17] but provides a better insight into the electricity prices.

VIII. ACKNOWLEDGEMENTS

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X. APPENDIX

Bilateral or Over The Counter (OTC) trade accounts for the largest volume in electricity trading in the Netherlands. Besides bilateral trade, trading can be done on three organized markets:
- Day-Ahead Market (APX DAM);
- Adjustment Market (APX); and
- Regulating Power Market.

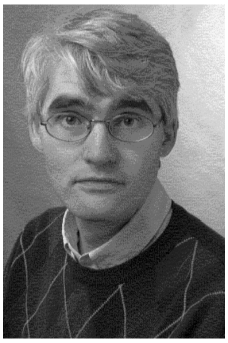
The Amsterdam Power Exchange (APX) started its operation in May 1999 with a Day Ahead Market (DAM) for electricity. The DAM is based on a two-sided auction model and offers hourly contracts, one day ahead of delivery. A price cap of 1600 €/MWh is used in the DAM. The APX DAM has a block-bidding functionality: participants can offer a certain volume (MW) for a block of hours (e.g. 10 hours) at a certain price (e.g. their variable costs). APX DAM will now match the block bid if the average market clearing price (MCP) over the 10 hours is above the offered price. If so, the block bid will be in the matching results for all 10 hours, even if in one (or some) hour(s) the MCP is below the offered price. APX DAM applies a matching for each individual hour and incorporates the block volume at the minimum price (0.01 Eur/MWh).

An adjustment market opened in February 2001. This adjustment market is based on continuous trade and offers market participants the possibility to avoid unexpected imbalances. This market closes a few hours before actual delivery. Market participants can place offers and bids from their portfolio of power plants and contracts and are therefore not directly related to single power plants. There is no central scheduling and dispatch of power plants by a market or System Operator. Before 2001, market prices were dominated by the so-called 'protocol', a framework contract between the four Dutch genco's and the disco's.

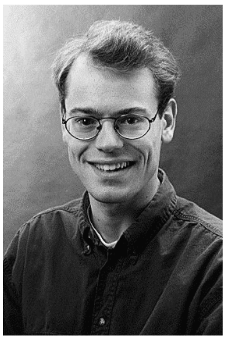
The number of APX participants currently amounts to 36. The total traded volume at the APX DAM was 8.24 TWh in 2001 (and 4.62 TWh in 2000). This accounts for about 9% of the total consumption in the Netherlands. Import plays an important factor and has almost doubled to 20 TWh per year since the opening of the market.

Since January 2001, TenneT (the Dutch Transmission System Operator) operates a regulating power market. TenneT uses this market during real-time operation to counteract imbalances between generation and demand for the Dutch system as a whole. The resulting clearing price is used to settle all actual imbalances per individual market party.

XI. Biographies



Adriaan J.P. de Lange was born in Alkmaar, the Netherlands on October 21, 1957. He obtained his M.Sc. in Electrical Engineering from the Delft University of Technology in 1986. In 1988, after conscription, he started working in the power industry. First as a mechanic and tester, later as an engineer and finally as project manager, mostly with 'Siemens AG Bereich KWU', building fossil power plants in the Netherlands and abroad. He received a Ph.D. for research on 'Three Phase Synthetic Testing of HV Circuit Breakers' at the Delft University of Technology in 2000. He received an MBA at Webster University (campus Leiden) in the same year. Currently dr. De Lange is working as a technical and business consultant for the power industry.



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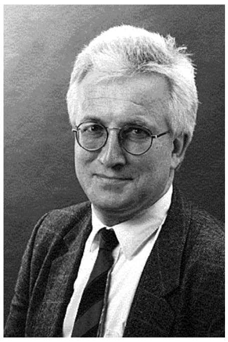

Lou van der Sluis was born in Geervliet, the Netherlands on July 10, 1950. He obtained his M.Sc. in Electrical Engineering from the Delft University of Technology in 1974. He joined the KEMA High Power Laboratory in 1977 as a test engineer and was involved in the development of a data acquisition system for the High Power Laboratory, computer calculations of test circuits and the analysis of test data by digital computer. In 1990 he became a part-time professor and since 1992 he has been employed as a full-time professor at the Delft University of Technology in the Electrical Power Systems Department. Prof. Van der Sluis is a senior member of IEEE and convener of WG CC-03 of Cigre and Cired to study the transient recovery voltages in medium and high voltage networks.